2.5 Postulates and Paragraph Proofs

Determine whether each statement is **always**, **sometimes**, or **never** true. Explain.

24. There is exactly one plane that contains noncollinear points A, B, and C.

**SOLUTION:**
Postulate 2.2 states that through any three noncollinear points, there is exactly one plane. Therefore, the statement is **always** true.
For example, plane \( K \) contains three noncollinear points.

**ANSWER:**
Always; Postulate 2.2 states that through any three non-collinear points, there is exactly one plane.

25. There are at least three lines through points \( J \) and \( K \).

**SOLUTION:**
Postulate 2.1 states through any two points, there is exactly one line. Therefore, the statement is **never** true.

**ANSWER:**
Never; Postulate 2.1 states through any two points, there is exactly one line.

26. If points \( M, N, \) and \( P \) lie in plane \( X \), then they are collinear.

**SOLUTION:**
The points do not have to be collinear to lie in a plane. Therefore, the statement is **sometimes** true.

**ANSWER:**
Sometimes; the points do not have to be collinear to lie in a plane.
27. Points X and Y are in plane Z. Any point collinear with X and Y is in plane Z.

**SOLUTION:**
Postulate 2.5 states if two points lie in a plane, then the entire line containing those points lies in that plane. Therefore, the statement is *always* true. In the figure below, points VWXY are all on line n which is in plane Z. Any other point on the line n will also be on plane Z.

![Diagram of points and lines](image)

**ANSWER:**
Always; Postulate 2.5 states if two points lie in a plane, then the entire line containing those points lies in that plane.

28. The intersection of two planes can be a point.

**SOLUTION:**
Postulate 2.7 states if two planes intersect, then their intersection is a line. Therefore, the statement is *never* true.

![Diagram of planes intersecting](image)

**ANSWER:**
Never; Postulate 2.7 states if two planes intersect, then their intersection is a line.
29. Points $A$, $B$, and $C$ determine a plane.

**SOLUTION:**
The points must be non-collinear to determine a plane by postulate 2.2. Therefore, the statement is *sometimes* true.

Three non-collinear points determine a plane.  
Three collinear points determine a line.

30. **PROOF** Point $Y$ is the midpoint of $\overline{XZ}$. $Z$ is the midpoint of $\overline{YW}$. Prove that $\overline{XY} \cong \overline{ZW}$.

**SOLUTION:**
You are given midpoints for two segments, $\overline{XZ}$ and $\overline{YW}$. Use your knowledge of midpoints and congruent segments to obtain information about $\overline{XZ}$ and $\overline{YW}$, the segments that you are trying to prove congruent.

**Given:** Point $Y$ is the midpoint of $\overline{XZ}$.
$Z$ is the midpoint of $\overline{YW}$.

**Prove:** $\overline{XY} \cong \overline{ZW}$.

**Proof:** We are given that $Y$ is the midpoint of $\overline{XZ}$ and $Z$ is the midpoint of $\overline{YW}$. By the definition of midpoint, $\overline{XY} \cong \overline{YZ}$ and $\overline{YZ} \cong \overline{ZW}$. Using the definition of congruent segments, $XY = YZ$ and $YZ = ZW$. $XY = ZW$ by the Transitive Property of Equality. Thus, $\overline{XY} \cong \overline{ZW}$ by the definition of congruent segments.

**ANSWER:**
Given: Point $Y$ is the midpoint of $\overline{XZ}$.
$Z$ is the midpoint of $\overline{YW}$.

Prove: $\overline{XY} \cong \overline{ZW}$.

Proof: We are given that $Y$ is the midpoint of $\overline{XZ}$ and $Z$ is the midpoint of $\overline{YW}$. By the definition of midpoint, $\overline{XY} \cong \overline{YZ}$ and $\overline{YZ} \cong \overline{ZW}$. Using the definition of congruent segments, $XY = YZ$ and $YZ = ZW$. $XY = ZW$ by the Transitive Property of Equality. Thus, $\overline{XY} \cong \overline{ZW}$ by the definition of congruent segments.
2.4 Determine whether each statement is always, sometimes, or never true. Explain.
24. There is exactly one plane that contains three coplanar lines. Sometimes; three coplanar lines may have 0, 1, 2, or 3 points of intersection, as shown in the figures below.

SOLUTION:
Three coplanar lines have two points of intersection.

ANSWER:
If the points were collinear, never; if the points were noncollinear, sometimes.

ANSWER:
Sometimes; the points do not have to be collinear to lie in a plane.

26. The intersection of two planes can be a point. Never; Postulate 2.7 states if two planes intersect, their intersection is a line.

SOLUTION:
Postulate 2.7 states if two planes intersect, their intersection is a line. Thus, the intersection of two planes cannot be a point.

27. Postulate 2.1 states through any two points there is exactly one line. Never; Postulate 2.2 states that through any three noncollinear points there is exactly one plane.

SOLUTION:
Postulate 2.2 states that through any three noncollinear points there is exactly one plane. Therefore, the statement is never true.

29. Postulate 2.4; a plane contains at least three noncollinear points.

SOLUTION:
Identify and equations to obtain information about the number of bikes and skateboards.

From the given information, there are a total of 11 bikes and skateboards, so if \( b \) represents bikes and \( s \) represents skateboards, \( b + s = 11 \). The equation can also be written \( s = 11 - b \). There are a total of 36 wheels, so \( 2b + 4s = 36 \), since each bike has two wheels and each skateboard has four wheels. Substitute the equation \( s = 11 - b \) into the equation \( 2b + 4s = 36 \) to eliminate one variable, resulting in \( 2b + 4(11 - b) = 36 \). Simplify the equation to \( 2b + 44 - 4b = 36 \) and solve to get \( b = 4 \). If there are 4 bikes, there are 11 - 4, or 7 skateboards. Therefore, there are 4 bikes and 7 skateboards.

ANSWER:
Sample answer: From the given information, there are a total of 11 bikes and skateboards, so if \( b \) represents bikes and \( s \) represents skateboards, \( b + s = 11 \). The equation can also be written \( s = 11 - b \). There are a total of 36 wheels, so \( 2b + 4s = 36 \), since each bike has two wheels and each skateboard has four wheels. Substitute the equation \( s = 11 - b \) into the equation \( 2b + 4s = 36 \) to eliminate one variable, resulting in \( 2b + 4(11 - b) = 36 \). Simplify the equation to \( 2b + 44 - 4b = 36 \) and solve to get \( b = 4 \). If there are 4 bikes, there are 11 - 4, or 7 skateboards. Therefore, there are 4 bikes and 7 skateboards.
In the figure, $\overline{CD}$ and $\overline{CE}$ lie in plane $P$ and $\overline{DH}$ and $\overline{DJ}$ lie in plane $Q$. State the postulate that can be used to show each statement is true.

34. Points $C$ and $B$ are collinear.

**SOLUTION:**
Identify $C$ and $B$ in the figure. If points $C$ and $B$ are collinear, then a line can be drawn through the two points. Postulate 2.1 states that through any two points, there is exactly one line.

**ANSWER:**
Postulate 2.1; through any two points, there is exactly one line.
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37. Points $D$ and $F$ are collinear.

**SOLUTION:**
Locate points $D$ and $F$. Postulate 2.1 states that through any two points, there is exactly one line. Therefore, you can draw a line through points $D$ and $F$.

**ANSWER:**
Postulate 2.1; through any two points, there is exactly one line.


**SOLUTION:**
Identify plane $Q$ and locate the points on it.
Postulate 2.4 states that a plane contains at least three noncollinear points.
Plane $Q$ contains the points $C$, $H$, $D$, and $J$.

**ANSWER:**
Postulate 2.4; a plane contains at least three noncollinear points.

41. Plane $P$ and plane $Q$ intersect at $\overrightarrow{CD}$

**SOLUTION:**
Identify plane $P$ and plane $Q$ and locate $\overrightarrow{CD}$.
Postulate 2.7 states that if two planes intersect, then their intersection is a line. Thus $\overrightarrow{CD}$ is the line of intersection of plane $P$ and plane $Q$.

**ANSWER:**
Postulate 2.7; if two planes intersect, then their intersection is a line.
42. **CCSS ARGUMENTS** Roofs are designed based on the materials used to ensure that water does not leak into the buildings they cover. Some roofs are constructed from waterproof material, and others are constructed for watershed, or gravity removal of water. The pitch of a roof is the rise over the run, which is generally measured in rise per foot of run. Use the statements below to write a paragraph proof justifying the following statement: The pitch of the roof in Den’s design is not steep enough.

- Waterproof roofs should have a minimum slope of \( \frac{1}{4} \) inch per foot.
- Watershed roofs should have a minimum slope of 4 inches per foot.
- Den is designing a house with a watershed roof.
- The pitch in Den’s design is 2 inches per foot.

**SOLUTION:**

Den is designing a watershed roof, so the minimum pitch for a waterproof roof are irrelevant to the question. We need to compare the pitch of Den's watershed roof with the minimum pitch for watershed roofs.

Sample answer: Since Den is designing a watershed roof, the pitch of the roof should be a minimum of 4 inches per foot. The pitch of the roof in Den’s design is 2 inches per foot, which is less than 4 inches per foot. Therefore, the pitch of the roof in Den’s design is not steep enough.

**ANSWER:**

Sample answer: Since Den is designing a watershed roof, the pitch of the roof should be a minimum of 4 inches per foot. The pitch of the roof in Den’s design is 2 inches per foot, which is less than 4 inches per foot. Therefore, the pitch of the roof in Den’s design is not steep enough.
REASONING Determine if each statement is *sometimes*, *always*, or *never* true. Explain your reasoning or provide a counterexample.

48. Through any three points, there is exactly one plane.

**SOLUTION:**

If the points were non-collinear, there would be exactly one plane by Postulate 2.2 shown by Figure 1.

![Figure 1](image1)

If the points were collinear, there would be infinitely many planes. Figure 2 shows what two planes through collinear points would look like. More planes would rotate around the three points. Therefore, the statement is *sometimes* true.

![Figure 2](image2)

**ANSWER:**

Sometimes; if the points were noncollinear, there would be exactly one plane by Postulate 2.2 shown by Figure 1. If the points were collinear, there would be infinitely many planes. Figure 2 shows what two planes through collinear points would look like. More planes would rotate around the three points.
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49. Three coplanar lines have two points of intersection.

**SOLUTION:**
Three coplanar lines, may or may not cross. If they cross, there can be 1, 2 or 3 points of intersection. Thus three coplanar lines may have 0, 1, 2, or 3 points of intersection, as shown in the figures below.

```
0 points  1 point
```

```
2 points  3 points
```

**ANSWER:**
Sometimes; three coplanar lines may have 0, 1, 2, or 3 points of intersection, as shown in the figures below.

```
0 points  1 point
```

```
2 points  3 points
```