1.3 Basic Functions

- **Identity Fn**: \( f(x) = x \)
- **Squaring Fn**: \( f(x) = x^2 \)
- **Cubing Fn**: \( f(x) = x^3 \)
- **Reciprocal Fn**: \( f(x) = \frac{1}{x} \)
- **Square Root Fn**: \( f(x) = \sqrt{x} \)
- **Exponential Fn**: \( f(x) = e^x \)
- **Natural Logarithm Fn**: \( f(x) = \ln x \)
- **Sine Fn**: \( f(x) = \sin x \)
- **Cosine Fn**: \( f(x) = \cos x \)
- **Absolute Value Fn**: \( f(x) = |x| \)
- **Greatest Integer Fn**: \( f(x) = \text{int}(x) \)
- **Logistic Fn**: \( f(x) = \frac{1}{1 + e^{-x}} \)
Find the domain of each basic function.
Which of the functions have points of discontinuity?
Which functions are bounded (above and below)?
Which functions are even?
Graph the function: \( f(x) = \sin(x) + 5 \)

a. On what interval, if any, is the function increasing? decreasing?

b. Is the function odd, even, or neither?

c. Give the function's extrema, if any.

d. How does the graph relate to any of the 12 basic functions?
Piecewise Function: a function whose domain is divided into several parts and a different function rule is applied to each part.

\[
f(x) = \begin{cases} 
  x^2 & \text{if } x \leq 0 \\
  \sqrt{x} & \text{if } x > 0 
\end{cases}
\]
Basic Function: Identity

\[ f(x) = x \]

Domain: \((-\infty, \infty)\)
Range: \((-\infty, \infty)\)
Continuity: continuous
Increasing/Decreasing behavior: \(\text{incr} (-\infty, \infty)\)
Symmetry: origin (odd)
Boundedness: unbounded
Local Extrema: none
Horizontal Asymptotes: none
Vertical Asymptotes: none
End behavior:

\[ \lim_{x \to -\infty} x = -\infty \]
\[ \lim_{x \to \infty} x = \infty \]
Basic Function: **Squaring**

\[ f(x) = x^2 \]

- **Domain:** \((-\infty, \infty)\)
- **Range:** \([0, \infty)\)
- **Continuity:** continuous
- **Increasing/Decreasing behavior:** \(\text{decr } (-\infty, 0] \text{ incr } [0, \infty)\)
- **Symmetry:** \(y\)-axis (even)
- **Boundedness:** unbounded / bounded below
- **Local Extrema:** abs. min \((0, 0)\)
- **Horizontal Asymptotes:** none
- **Vertical Asymptotes:** none
- **End behavior:**
Basic Function: Cubing

\[ f(x) = x^3 \]

Domain: \((-\infty, \infty)\)

Range: \((-\infty, \infty)\)

Continuity: Continuous

Increasing/Decreasing behavior: \(\text{inc} \ (-\infty, \infty)\)

Symmetry: Origin (Odd)

Boundedness: Unbounded

Local Extrema: None

Horizontal Asymptotes: None

Vertical Asymptotes: None

End behavior:
Basic Function: \( f(x) = \frac{1}{x} \)

- **Domain:** \((-\infty, 0) \cup (0, \infty)\)
- **Range:** \((-\infty, 0) \cup (0, \infty)\)
- **Continuity:** Infinite discontinuity
- **Increasing/Decreasing behavior:** \(\text{dec. } (-\infty, 0) \cup (0, \infty)\)
- **Symmetry:** Origin (odd)
- **Boundedness:** Unbounded
- **Local Extrema:** None
- **Horizontal Asymptotes:** \(y = 0\)
- **Vertical Asymptotes:** \(x = 0\)
- **End behavior:**
Basic Function: \( \text{Square Root} \)

\[ f(x) = \sqrt{x} \]

Domain: \([0, \infty)\)

Range: \([0, \infty)\)

Continuity: continuous

Increasing/Decreasing behavior: incr \((0, \infty)\)

Symmetry: none

Boundedness: unbounded / bounded below

Local Extrema: abs. min \((0, 0)\)

Horizontal Asymptotes: none

Vertical Asymptotes: none

End behavior:
Basic Function: Exponential

\( f(x) = e^x \)

Domain: \((-\infty, \infty)\)

Range: \((0, \infty)\)

Continuity: continuous

Increasing/Decreasing behavior: incr \((-\infty, \infty)\)

Symmetry: none

Boundedness: unbounded / bounded below

Local Extrema: none

Horizontal Asymptotes: \(y = 0\)

Vertical Asymptotes: none

End behavior:
Basic Function: **Natural Logarithm**

\[ f(x) = \ln x \]

- **Domain:** \((0, \infty)\)
- **Range:** \((-\infty, \infty)\)
- **Continuity:** Continuous
- **Increasing/Decreasing behavior:** \(\text{incr. } (0, \infty)\)
- **Symmetry:** None
- **Boundedness:** Unbounded
- **Local Extrema:** None
- **Horizontal Asymptotes:** None
- **Vertical Asymptotes:** \(x = 0\)
- **End behavior:**
Basic Function: Sine

\[ f(x) = \sin(x) \]

Domain: \((\infty, \infty)\)
Range: \([-1, 1]\)
Continuity: Continuous
Increasing/Decreasing behavior: alt. incr./decr.
Symmetry: origin (odd)
Boundedness: bounded
Local Extrema: abs. maxs \((\pm 1)\) abs. mins \((\pm 1)\)
Horizontal Asymptotes: none
Vertical Asymptotes: none
End behavior:
Basic Function: \( \cos(x) \)

- **Domain:** \((-\infty, \infty)\)
- **Range:** \([-1, 1]\)
- **Continuity:** \(\text{cont.}\)
- **Increasing/Decreasing behavior:** \(\text{alt. incr/decr}\)
- **Symmetry:** \(y\)-axis (even)
- **Boundedness:** bounded
- **Local Extrema:** abs. maxs \((at \ 1)\) abs. mins \((at \ -1)\)
- **Horizontal Asymptotes:** none
- **Vertical Asymptotes:** none
- **End behavior:**
Basic Function: Absolute Value

\[ f(x) = |x| \]

Domain: \((-\infty, \infty)\)

Range: \([0, \infty)\)

Continuity: \text{continuous}

Increasing/Decreasing behavior: \(\text{incr } [0, \infty) \text{ decr } (-\infty, 0]\)

Symmetry: \(y\)-axis (even)

Boundedness: \(\text{unbounded} / \text{bounded below}\)

Local Extrema: \(\text{abs. min } (0, 0)\)

Horizontal Asymptotes: \(\text{none}\)

Vertical Asymptotes: \(\text{none}\)

End behavior: \(\text{up right, up left}\)
Basic Function: Greatest Integer

\[ f(x) = \text{int}(x) \]

\[
\begin{align*}
\text{int}(5.9) &= 5 \\
\text{int}(5.1) &= 5 \\
\text{int}(5.0) &= 5 \\
\text{int}(4.9) &= 4
\end{align*}
\]

Domain: \((-\infty, \infty)\)

Range: all integers

Continuity: jump discontinuity

Increasing/Decreasing behavior: \(\text{constant} / \text{increasing}\)

Symmetry: none

Boundedness: unbounded

Local Extrema: none

Horizontal Asymptotes: none

Vertical Asymptotes: none

End behavior: ___________________________
Basic Function: Logistic

\[ f(x) = \frac{1}{1 + e^{-x}} \]

Domain: \((-\infty, \infty)\)

Range: \((0, 1)\)

Continuity: continuous

Increasing/Decreasing behavior: incr \((-\infty, \infty)\)

Symmetry: about \((0, 0.5)\)

Boundedness: bounded

Local Extrema: none

Horizontal Asymptotes: \(y = 0\) and \(y = 1\)

Vertical Asymptotes: none

End behavior: ________________