3.4 Direct Variation

Definition: A direct variation is a linear function that can be written in the form

$$y = kx \qquad \text{where } k \neq 0$$

k: constant of variation
(or constant of proportionality)

When the dependent variable
depends solely on the value of the independent variable,
it is a direct variation.

**The graph of a direct variation will always pass through the origin!

$y = kx + 0$

Example:

You have a job babysitting for $7 per hour.

The amount of money you make

varies directly with the number of hours you work.

The equation that relates your total pay to the number of hours you work is

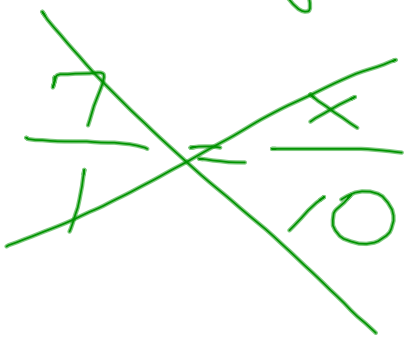
$$y = 7x$$

$$\frac{7}{1} = \frac{x}{10}$$

Not - Proportional (Not a direct variation)

$7 per hour + $5 tip

$$y = 7x + 5$$



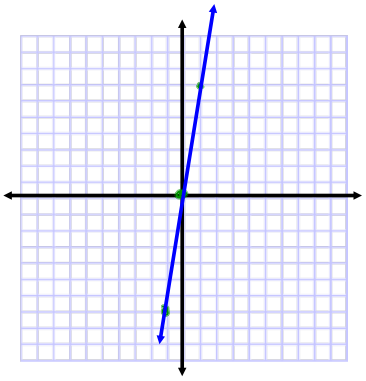
Does the equation represent a direct variation?
If so, what is the constant of variation?

***Graph the direct variations!**

$y = 7x$

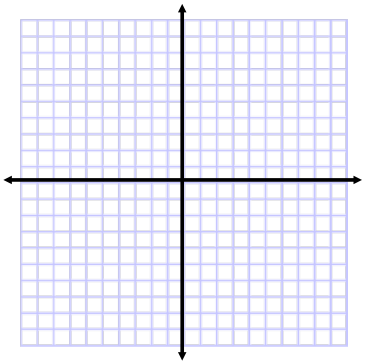
yes!

$k = 7$



$y = 4x - 10$

no.



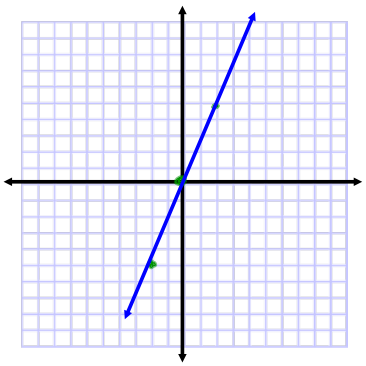
$5x - 2y = 0$

$+2y \quad +2y$

$\frac{5x}{2} = \frac{2y}{2}$

$\frac{5}{2}x = y$

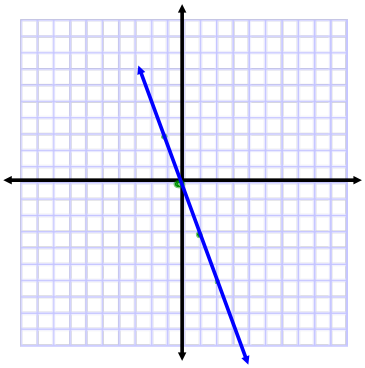
yes! $k = \frac{5}{2}$



$x + y = 7$

$-x \quad -x$

$y = -x + 7$



$\frac{-8y}{-8} = \frac{24x}{-8}$

$y = -3x$

yes! $k = -3$

Examples:

Suppose y varies directly as x.
When x = 5, y = 20.
Write an equation that relates x and y

$y = kx$

$\frac{20}{5} = \frac{k \cdot 5}{5}$

$4 = k$

$y = 4x$

Now find the value of y when x = 20

$y = 4x$

$y = 4(20)$

$y = 80$

Suppose y varies directly as x.
When x = 21, y = 3.
Write an equation that relates x and y

$\frac{3}{21} = \frac{k \cdot 21}{21}$

$\frac{3}{21} = k$

$\frac{1}{7} = k$

$y = \frac{1}{7}x$

Now find the value of y when x = 14

$y = \frac{1}{7}x$

$y = \frac{1}{7}(14)$

$y = 2$

Tell whether the relationship is a direct variation.

x	y
10	2
15	3
20	4

+5, +1, +5, +1

$\frac{y}{x} = \frac{kx}{x}$

$\frac{y}{x} = k$

Check $\frac{y}{x}$

$\frac{2}{10} = \frac{3}{15} = \frac{4}{20}$

yes! because $\frac{y}{x}$ is constant

x	4	8	12
y	16	20	24

4, 4, 4, 4

Check $\frac{y}{x}$

$\frac{16}{4} \stackrel{?}{\neq} \frac{20}{8} \neq \frac{24}{12}$

No because $\frac{y}{x}$ is not constant

x	-3	-6	-9
y	18	36	54

-3, -3, 18, 18

check $\frac{y}{x}$

$\frac{18}{-3} = \frac{36}{-6} = \frac{54}{-9}$

yes! $\frac{y}{x}$ is constant

x	8	0	-5
y	13	5	0

The amount of money you get paid varies directly with the number of hours you work.
When you work 17 hours, you get paid $80.41.

Write a direct variation equation that relates your pay and the number of hours you work.

$$y = kx$$

$$\frac{80.41}{17} = \frac{k \cdot 17}{17}$$

$$4.73 = k$$

$$y = 4.73x$$

Now find your pay when you work 20 hours.

$$y = 4.73x$$
$$y = 4.73(20)$$
$$y = \$94.60$$

Now find the number of hours you need to work to get paid $137.17

$$y = 4.73x$$

$$\frac{137.17}{4.73} = \frac{4.73x}{4.73}$$

$$29 \text{ hrs} = x$$