8.1 Similar Right Triangles

With a 3 x 5 card, use a straightedge to form 3 right triangles as seen below. Cut the three triangles out.

![Diagram of similar right triangles]

**Theorem:** The altitude to the hypotenuse of a right triangle forms two triangles that are similar to each other and to the original triangle.
\[ \triangle ADC \sim \triangle ACB \sim \triangle CDB \]

Now pull your triangles apart and arrange them so corresponding parts are in the same position:

\[ \triangle ADC \sim \triangle ACB \sim \triangle CDB \]

\[
\begin{align*}
\frac{AD}{AC} &= \frac{AC}{AB} \\
\frac{AD}{CD} &= \frac{CD}{BD} \\
\frac{AB}{CB} &= \frac{CB}{BD}
\end{align*}
\]
Look at the first proportion we made:

\[
\frac{AD}{AC} = \frac{AC}{AB}
\]

The \textit{means} of the proportion are the same number. That number is the \textit{geometric mean} of the extremes.

The \textit{geometric mean} of two positive numbers is the positive square root of their product.

\[
\frac{a}{x} = \frac{x}{b}
\]

\[
x^2 = ab
\]

\[
x = \sqrt{ab}
\]

Find the geometric mean of 2 and 8

\[
x = \sqrt{2 \cdot 8}
\]

\[
x = \sqrt{16}
\]

\[
x = 4
\]

Find the geometric mean of 10 and 30

\[
x = \sqrt{10 \cdot 30}
\]

\[
x = \sqrt{300}
\]

\[
x = \sqrt{100 \cdot 3}
\]

\[
x = 10\sqrt{3}
\]
**Corollary:** The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the two segments of the hypotenuse.

\[ h^2 = xy \]

**Corollary:** The length of a leg of a right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse adjacent to that leg.

\[ a^2 = xc \]
\[ b^2 = yc \]
1. 

\[
\begin{align*}
    x^2 &= 6 \cdot 10 \\
    x^2 &= 60 \\
    x &= \sqrt{60} \\
    x &= \sqrt{4 \sqrt{15}} \\
    x &= 2 \sqrt{15}
\end{align*}
\]
2. \[ y^2 = 5 \cdot 13 \]
\[ y = \sqrt{65} \]
3.

\[ 16^2 + 5.75^2 = x^2 \]

\[ 289.0625 = x^2 \]

\[ 17 \approx x \]

\[ 17^2 = 5.75h \]

\[ 289 = 5.75h \]

\[ 50.26 \approx h \]
4.

\[ \begin{align*}
\text{If } y = 4.13, \quad y^2 &= 5.2 \\
\text{Then, } 6^2 + 4^2 &= y^2 \\
36 + 16 &= \sqrt{52} \\
52 &= y \\
\sqrt{52} &= y \\
\sqrt{4 \cdot \sqrt{13}} &= y \\
2\sqrt{13} &= y
\end{align*} \]

\[ \begin{align*}
x &= 4.9 \\
x^2 &= 36 \\
x &= 6
\end{align*} \]

\[ \begin{align*}
z &= \sqrt{117} \\
z &= \sqrt{9} \cdot \sqrt{13} \\
z &= 3\sqrt{13}
\end{align*} \]
5.

\[ 9^2 = x \cdot 15 \]

\[ 81 = 15x \]

\[ 5.4 = x \]

\[ y = 15 - 5.4 \]

\[ y = 9.6 \]
\[w = 25 - 9 = 16\]
\[x^2 = 9(25)\]
\[x^2 = 225\]
\[x = 15\]
\[y^2 = 9(16)\]
\[y^2 = 144\]
\[y = 12\]

\[z^2 = 16(25)\]
\[z^2 = 400\]
\[z = 20\]